Stages in Multiplicative Thinking

Recognizing the structure of a pattern—initially, a simple repetition—is central238 Preschoolers'Mathematical Patterning to the notion of unit of repeat and the development of composite units (Steffe, 1994). Thus, patterning, even at a primitive level of skip counting, can play an important role in the development of multiplicative reasoning (Mulligan & Mitchelmore, 1997; Nunes, Bryant, Burman, Bell, Evans, & Hallett, 2009). The early patterning experiences of young children often involve simple repetition using one variable (e.g., blue, red, blue, red) (Papic, 2007). This may account for difficulties that older students face in recognizing and generalizing patterns and relationships. Often, the teaching of patterning skills focuses on additive thinking, rather than on multiplicative thinking, which is necessary for developing composite units in complex repetitions, and on constructing growing patterns and functional relationships (Mulligan & Mitchelmore, 1997). A proportion is a relationship between quantities; that is, how two quantities are related multiplicatively (Small, 2010). Thinking of 8 as two 4s, instead of as 2+6 or 4+4 highlights the difference between multiplicative thinking and additive thinking. In fact, proportional thinking involves the ability to generate a missing component through multiplicative reasoning, regardless of the numerical aspects of the problem situation. A student who recognizes that a person who runs 3 km in 4 minutes finishes a 15 km run faster than one who runs 5 km in 8 minutes is thinking proportionally. Students need to look at a relationship between numbers instead of looking at one number in isolation. Subsequently, proportional reasoning requires the mental storage and processing of several bits of information as it involves a sense of co-variation and multiple interpretation and comparisons of both the meaning of two individual ratios and the comparison of ratios (Lesh, Post, & Behr, 1988). From ABACUS (KKZ & MLK)

Jacob & Willis (200?)	**One to One Counting**: One to one correspondence without connecting to total amount	**Additive Composition**: Count as a permanent indicator of the quantity or amount; and that rearrangements do not change amounts or quantity In multiplicative terms, the consequence of this thinking is that if children recognise equal groups they are in a position to take advantage of the groups to count more efficiently using skip counting and repeated addition. However, they may still need to lay the items in the groups out before they skip count or repeatedly add to find out how many. They do not yet understand that groups themselves can be counted. Their focus is on the multiplicand and they do not understand the role of the multiplier.	**Many to One Counters:** When children understand that groups can be counted and that they can keep track of two things at once — the number of groups and the total of the number in each group — they are in this phase. They can hold two numbers in their head at once and double count. Children now know that they can represent one group and count repetitions of that same group.	**Multiplicative Relations:** When children come to know that multiplicative situations involve three aspects: groups of equal size (a multiplicand), numbers of groups (the multiplier), and a total amount (the product), and can coordinate the grouping structure in both multiplication and division problems prior to carrying out the count, they are in this phase.	**Operate the Operator:** When children can operate on variables in algebraic situations and operate on operations they are in this phase. It would mean that they could multiply the multiplier in the way described by Schmidt and Weiser (1995) in the multiplication problem type they call the structure of composition of operators. They give an example of this type of problem: During the first year of life Otto the elephant triple his weight at birth. In his second year he doubles his weight. What multiple of his weight has he got at the end of his second year of life? (p. 60) Also among other things it would mean that they could flexibly use factors in order produce a multiplication number sentence that was easier to calculate.
Model of the Development of Early Multiplication and Division Knowledge – Wright, Martlan & Stafford (2006).	**Level 1: Initial Grouping** "Uses perceptual counting (that is, by ones) to establish the numerosity of a collection of equal groups, to share items into groups of a given size (quotitive sharing), and to share items into a given number of groups (partitive sharing). -items are visible and student counts by ones." p. 120-121	**Level 2: Perceptual Counting in Multiples** "Uses a multiplicative counting strategy to count visible items arranged in equal groups. -counting strategies are more advanced than those used in level 1. These counting strategies fall into three categories - Perceptual Rhythmic Counting, Perceptual Double Counting, Perceptual Skip Counting." p. 122	**Level 3: Figurative Composite Grouping** "Uses a multiplicative counting strategy to count items arranged in equal groups in cases where individual items are not visible." -child has developed counting strategies which do not rely on items being visible and which do not involve counting by ones. E.g., if the child is presented with four groups of three counters, where each group is separately screened, the child may use skip counting by threes to determine the number of counters in all (three, six, nine, twelve). From the child's perspective, each of the four screens symbolizes a collection of three items but the individual items are not visible." p. 123	**Level 4: Repeated Abstract Composite Grouping** "Counts composite units in repeated addition or subtraction, that is, uses the composite units a specified number of times. -child has constructed a conceptual structure labeled an 'abstract composite unit' in which the child is simultaneously aware of both the composite and unitary aspects of three for example. The child can use repeated addition to solve multiplication tasks and repeated subtraction to solve division tasks, and can do so in the absence of visible or screened items. On a multiplicative task involving six groups of three items, in which each group is separately screened, the child is aware of each group as an abstract composite unit." p. 123-124	**Level 5: Multiplication and Divsion as Operations** "Can regard both the number in each group and the number of groups as a composite unit. Can immediately recall or quickly derive many of the basic facts for multiplication and division.- the child can coordinate two composite units in the context of multiplication or division. In a task such as six threes or six groups of three, for example, the child is aware of both six and three as abstract composite units, whereas at Level 4 the child is aware of three as an abstract composite unit but is not aware of six as an abstract composite unit. The child at Level 5 can immediately recall or quickly derive many of the basic facts of multiplication and division and may use multiplication facts to derive division facts. At level 5, the commutative principle of multiplication (e.g., 5x3=3x5) and the inverse relationship between multiplication and division are within the child's zone of proximal development. Thus, for example, the child might be aware that six threes is the same as three sixes and might use 4x8=32 to work out 32/4." p.125

DRAFT

OUR TEAM's OBSERVED CONTINUUM	**Stage one: Additive: Anna, Keisha, Caleb, Karson** Everything visible – needs to see both nests and eggs Counting by ones Have a place holder for the multipliers (holds one but not the other) Anna Visualizing the multiplicand (multiplier was there but not filled in) – tiny step – Keisha Can solve with only one or the other (multiplier or multiplicand) visible Makes groups but counts by ones Kyra Karson	**Stage two: multiplicative thinking: Lilly, Mac, Madisynn, Simon** Some visual scaffolding – beyond the concrete Can hold both all the 3 things (quantity, groups, number in groups)	**Stage 3: Ready for proportional reasoning: Drake, Ben, Sarah** Flexible thinkers Can do the three things (multiplicative relations) Skip counting Visualizing with the multiplicand as a unit Using knowledge of operations to figure it out	**Stage 4: early proportional reasoning: Ryan** *Ryan (Grade 2)* Multiplicative relations Fluency with number No visuals required Thinking about term numbers (nests) and the eggs as outcome (shows as graph as well as in his explanation) *consider using this video for math camp	

DRAFT